TECHNICAL MEMORANDUMS

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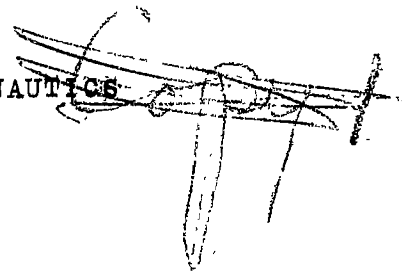

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DIAGRAMS FOR CALCULATION OF AIRFOIL LATTICES

By Albert Betz

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SUMMARY

The field for curved blades is represented by a vortex series with a vortex removed at the blade point. Further, an example of calculation of a curved blade from this series is given, whereby the necessary accuracy required of the different methods in practice is shown according to the case considered.

INTRODUCTION

In the new theory of impeller wheels, airfoil lattices (blade grids) have an important place (see reference 1,p.219, from which figs. 1 through 3 are taken); that is, for circularly arranged blades (centrifugal pump, fig. 1) or for helical sections of propeller wheels (propeller pumps, Kaplan turbines, fig. 2). This gives directly a straight airfoil lattice (fig. 3) and the former can be reduced to one by a simple conformal transformation ($\zeta = \ln Z$).

The size and shape of the blade grid obtained for fixed working conditions can be secured from the extensive data for single airfoils, if the following are considered: If an airfoil is removed from out of a grid, the streamlines of the flow at the point are already curved through the influence of the adjacent blades. If one puts a thin airfoil at this point, the shape (profile) of which coincides exactly with a streamline, obviously, little force is exerted on such a blade, as it certainly does not alter the shape of the streamline and it thus exerts little force on the fluid. A force occurs if the airfoil differs from the shape of the streamline, and the force and distribution of it depend essentially on the deviation. One can only approximately assume that the

*"Diagramme zur Berechnung von Flügelreihen." Ingenieur-Archiv, Bd. II, Heft 3, Sept. 1931, pp. 359-371.

action of two blades is equal if their deviation from the undisturbed streamlines is equal; that is, the action for use of this rule does not depend greatly on the initial form of the streamlines.*

It can only be approximately assumed that the action of two blades is equal if their deviation from the undisturbed stream is equal; that is, for this rule the action does not greatly depend on the form of the entering streamlines. The practice for a single blade can thus be used if this shape is altered to fit the basic flow. Moreover, as previously seen, the basic flow is obtained at the point by removing a blade, but without simultaneously changing the circulation distribution of the other blades.

If the blade chord is not great with respect to the spacing between blades, the resulting flow can be computed by replacing the blades with vortices of equal circulation. The mathematical form representing such a field by a vortex series is well known (reference 1, p. 238). The formula is inconvenient for practical use, otherwise we should break the field into a series for which a vortex is missing. We must therefore remove a vortex from the field, whence we get further difficulty. In the region of the vortex point concerned, that is, the region in which we are mainly interested, the function approaches infinity.

These troublesome calculations must not always be repeated, as the distribution of the important values in the region in question is given in figures 5 to 8. Because of symmetry it is sufficient to give only one quadrant. One must only notice the sign to find the quadrant. To facilitate their use a small supplementary drawing is given in the top right-hand corner of each plot, which gives information as to the sign of the single quadrant. Figure 4 shows the arrangement of the vortices with the essential dimensions and symbols.

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*Exact agreement with these assumptions is not obtained. Besides, appreciable variation occurs if the flow in which the blade is placed is accelerated or decelerated. (Grids with pressure rise or fall, pumps or turbines.) (See reference 2.) Also for strongly curved flows (vanes with large change of direction) are variations to be expected. If one is sufficiently familiar with it, these variations are immaterial to the satisfactory application of the given diagrams.

The vortex spacing is a, the circulation of each vortex Γ. The diagram uses the dimensionless ratios x/a, y/a,

$$u^* = u\,\frac{a}{\Gamma},\quad v^* = v\,\frac{a}{\Gamma},\quad w_n^* = w_n\,\frac{a}{\Gamma},\quad \Phi^* = \frac{\Phi}{\Gamma},\quad \Psi^* = \frac{\Psi}{\Gamma}$$

For the diagrams, vortex series extending farther than $\frac{\Gamma}{2a}$ on one side and $-\frac{\Gamma}{2a}$ on the other parallel to the grid are taken as basis. The component normal to the grid is zero. In the practical case the grid may, in general, have any arbitrary inflow c_1, having components u_1 and v_0 (fig. 10). For example, the blade series causes a deflection of the flow so that at unit distance behind the grid the velocity c_2 has components u_2 and v_0. (On the basis of continuity the component v_0 cannot change.) The generalized flow is obtained from the one described through superposition of a parallel flow with velocity c_0, the components of which are $u_m = \frac{u_1 + u_2}{2}$ and v_0. To obtain the desired deflection, the change in u_1 and u_2, the circulation about each blade must be considered.

$$\Gamma = (u_1 - u_2)\,a \tag{1}$$

The velocity at any point then has, if the corresponding point in the diagram has the components $u^* = u\,\frac{a}{\Gamma}$ and $v^* = v\,\frac{a}{\Gamma}$, the components

$$\left.\begin{aligned} u' &= \frac{u_1 + u_2}{2} + (u_1 - u_2)\,u^* \\ v' &= v_0 + (u_1 - u_2)\,v^* \end{aligned}\right\} \tag{2}$$

The pressure rise (negative = pressure fall) is known (reference 1, p. 231).

$$p_2 - p_1 = \frac{\rho}{2}(u_1{}^2 - u_2{}^2) - p' \tag{3}$$

where p' is the loss in the lattice. The ~~pressure~~ Force exerted by a blade has the components

$$\left.\begin{aligned} P_x &= \rho\, v_0\, (u_1 - u_2)a \\ P_y &= (p_2 - p_1)a \end{aligned}\right\} \tag{4}$$

To explain the use of the plots, we will only determine the proper shape and placing of the blades in a series, when the series must produce a given effect. We proceed as follows: We do not at first consider the influence of adjacent blades, which is accurate if the blade chord is very small compared to the grid spacing. (1st approximation). We then compute the change by means of the charts considering the influence of the neighboring blades. We next imagine that the neighboring blades are replaced by a single line vortex, as in the diagrams. This gives satisfactory results for most practical cases (2d approximation). We can immediately drop this simplified approach and consider the chordwise distribution of circulation along the blade, where we can also use the diagrams (3d approximation).

CALCULATION OF BLADES WITHOUT CONSIDERING THE DISTURBANCE DUE TO THE ADJACENT BLADES

The spacing a is, in general, obtained from considerations of construction. The inflow velocity is given by its components v_0 and u_1. The blade series will increase the velocity u_1 to u_2 (turbine blades). Instead of the velocity change the force in the grid direction (tangential force) P_x can be used. From the governing relation $(u_2 - u_1)$ can be easily computed. (The pressure drop $(p_1 - p_2)$ or the force perpendicular to the grid is required; thus the pressure loss p' must be estimated next on the basis of corrected airfoil data.) The circulation Γ is given by equation (1) from $(u_2 - u_1)$.

In aeronautical literature are given values of c_a (lift coefficient) and c_w (drag coefficient) depending on the angle of attack α for wing profiles. Besides the so-called glide angle, the ratio $\frac{c_w}{c_a} = \epsilon$ plays a further part in the loss p', or the machine efficiency. The circulation Γ is related to the lift coefficient c_a of a blade in the lattice by

$$c_a \approx \frac{2\,\Gamma}{c_o t} \tag{5}$$

where t is the blade chord in the direction of flow. The hypothesis for this formula is that the velocity is not essentially changed in the region from the leading edge to the trailing edge of the blade. If this condition is not satisfied, the blade chord must be corrected, whereupon we return to figures 11 to 13. We will now select a suitable airfoil from those tested, for example, a plate with small camber, $\frac{\text{camber}}{\text{chord}} = 0.05$ (reference 3, p. 407.* Figure 9 gives test values for this airfoil.

In case cavitation or other considerations do not dictate a very low value of c_a, we choose c_a to give a satisfactory small value of the glide angle (low drag). In this case** for the airfoil chosen: $c_a = 0.7$ and $\alpha = 3^o$.

With this lift coefficient, and the magnitudes of u_1 and u_2 given by the problem, the necessary chord is fixed by equations (1) and (5).

$$\frac{t}{a} \approx \frac{2(u_1 - u_2)}{c_a\, c_o}$$

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* The values given there must be corrected to infinite aspect ratio. (Formulas given in Hütte, p. 402.)

**For pressure drop through the grid, one can probably pick a higher value of c_a for this case. (K. Christiani, see footnote p. 2 of this report.)

For example, from the problem we require $v_o = 2u_1$ and $u_2 = 3u_1$ thus,

$$\left.\begin{aligned} \frac{\Gamma}{a} &= u_2 - u_1 = 2u_1 \qquad u_m = \frac{u_1 + u_2}{2} = 2u_1 = \frac{\Gamma}{a} \\ c_o &= \sqrt{u_m^2 + v_o^2} = u_1\sqrt{2^2 + 2^2} = 2.82\ u_1 = 1.41\ \frac{\Gamma}{a} \end{aligned}\right\} \quad (7)$$

and thereby

$$\frac{t}{a} \approx \frac{2 \times 2}{0.7 \times 2.82} = 2.03 \quad (8)$$

If $\frac{t}{a}$ is very small, we can consider the flow in the region of the blades as uniform (1st approximation), and this flow has two components $u_m = \frac{u_1 + u_2}{2}$ and v_o.

The deflection velocity for the blades considered at the zero point of the diagram leave only the superimposed parallel flow. We must therefore arrange the blades with the proper angle of attack relative to this flow. In our example the superimposed flow has the components $u_m = 2u_1$ and $v_o = 2u_1$, thus its direction forms the angle $\beta = \tan^{-1} \frac{v_o}{u_m} = 45^\circ$ with the grid direction. The blade chord has an angle of attack of 3° with this direction; it thus makes an angle with the grid direction of $\delta = \beta - \alpha = 42^\circ$. The arrangement of the blades is shown in figure 10 according to the calculations. But it is intentionally emphasized that for the example the assumption that the blade must be small compared to the grid spacing, is not satisfactorily fulfilled, and for that reason the arrangement shown in figure 10 is not satisfactory.

INFLUENCE OF REPRESENTING ADJACENT BLADES BY VORTICES

Calculations based on Deflection Velocities

If the blade is not small compared to the grid spacing, the tail thus projects noticeably into the disturbed region of the adjacent blade. The flow direction there is different from that in the middle and we must adjust the blade curvature for this changed relation. For this calculation we can, in general, replace the adjacent airfoils by separate vortices (2d approximation). Since the actual blade occupies considerable space, we must fix a place in the blade to put the replacing vortex. It has been shown that the least error is made if the vortex is placed at the so-called center of pressure, that is, the point at which the resultant force cuts the blade chord. It lies, in general, at a distance s_1 between 0.25t and 0.5t from the leading edge. The moment coefficient c_m about the leading edge is usually given in the reports of airfoil research (fig. 9). Whence one can calculate

$$\frac{s_1}{t} = \frac{c_m}{c_a} \tag{9}$$

For our example $c_a = 0.7$ $c_m = 0.29$, thus $\frac{s}{t} = \frac{0.29}{0.70} = 0.41$. In figure 10 the x-axis is drawn through the profile center of pressure 0.41 t from the leading edge. The deflection velocities can be easily found for each point of the blade by use of diagrams 6 to 8. If the deflection is not very large, it is enough to find the relation to correct the blade at the leading and trailing edges, especially the latter. As the deflection becomes less, the position of the leading and trailing edges, computed from the deflection, approaches those for the basic undisturbed stream. For a considerable displacement of the edges, it can be obtained by repeated calculations, but, in general, this will not be greatly changed from the assumed position.

In our example the edges for the undisturbed flow lie on a straight line making an angle $\delta = 42^\circ$. The leading edge is removed from the zero point a distance

$s_1 = 0.41t = 0.89a$ ($t = 2.03a$ from equation (7)), and the trailing edge a distance $s_2 = 0.59t = 1.20a$.

The angular deviation of the flow follows from the deflection velocities u and v (charts 6 and 7) from the adjacent vortices. According to figure 11

$$\Delta\beta \simeq \tan\Delta\beta = \frac{v\cos\beta - u\sin\beta}{c_0 + u\cos\beta + v\sin\beta} \tag{10}$$

It should be noticed that u and v have different signs in the different quadrants, but β must always be less than 180°. For the leading edge of our example u and v ~~is~~ are negative. In many cases the deflection component $u\cos\beta + v\sin\beta \ll c_0$. In which case the calculations can be simplified if the deflection is not calculated for the blade itself, but rather for the point on the undisturbed streamline c_0 through the zero point making an angle β with the x-axis (fig. 10). This deviation component $v\cos\beta - u\sin\beta$ is identical with the component w_n given in chart 8, and we get

$$\Delta\beta \simeq \pm \frac{w_n}{c_0} \tag{11}$$

The positive sign applies to the entrance distance $y > 0$ and the negative sign to the exit $y < 0$. For our example we read from the chart ($\beta = 45^\circ$, $s_1 = 0.83a$) $w_n = -0.14 \frac{\Gamma}{a}$, and for the trailing edge ($\beta = 45^\circ$, $s_2 = 1.19a$) $w_n = 0.22 \frac{\Gamma}{a}$.

Since $c_0 = 1.41 \frac{\Gamma}{a}$ (see equation (7)), we get for the correction angle,

$$\left.\begin{array}{ll} \text{for the leading edge} & \Delta\beta \approx 7\tfrac{1}{2}^\circ \\ \text{for the trailing edge} & \Delta\beta \approx -7^\circ \end{array}\right\} \tag{12}$$

We must consider that the blade does not give an appreciable change in the velocity c_0. For a given wing the circulation is proportional to the velocity. To maintain constant circulation we must therefore reduce this velocity in inverse ratio to the blade speed and similarly increase it for lower speeds. The velocity at the leading edge in the example, terms in equation (9),

$$c_E = 0.74\ c_o$$

and at the trailing edge.

$$c_A = 1.26\ c_o$$

If we now assume that the velocity varies approximately linearly from the zero point to the edges, then the mean velocities for these distances are $\frac{c_o + c_E}{2}$ or $\frac{c_o + c_A}{2}$ and from the original equations, these velocity changes require changes in length of the separate blade elements at the inlet or exit edges of the distance s_1 or s_2, respectively, from the zero point. Where

$$\left.\begin{aligned} s'_1 &= s_1 \frac{2\ c_o}{c_o + c_E} = 1.15 \qquad s_1 = 0.96a \\ s'_2 &= s_2 \frac{2\ c_o}{c_o + c_A} = 0.885 \qquad s_2 = 1.05a \end{aligned}\right\} \qquad (13)$$

With these corrections we get the changes in the airfoil shown in figure 11, the proportional change in the tail point being clearly shown. Practically it is almost meaningless compared to the change in angle of the leading and trailing edges. It is noticed besides that for this change only the change in total chord of the airfoil has effect. If, therefore, as in the above case, the end points are displaced equally, there is scarcely any difference. To be sure this applies only when the streamline along which the profile lies is in some degree straight. For strongly curved streamlines (large velocity change in the lattice), the displacement of the profile itself plays an important part.

Calculation by Conformal Transformation

If, as in our example, the simple formula (11) does not satisfactorily calculate the necessary bend in the blade, it is then simple and accurate to obtain the curve by means of chart 5, in the following way (figs. 12 and 13).

If we draw the streamlines of the undisturbed flow superimposed on the deflected flow (fig. 5) to an equal scale, then streamlines of the undisturbed flow are straight lines, for our example lines at 45° to the x-axis. Since $c_0 = 1.41 \frac{\Gamma}{a}$, thus choosing the distance between these streamlines and numbering them so that streamline "zero" goes through the zero point and streamline "one" is displaced a distance $\frac{a}{1.41}$ figure 12 shows for the example the streamlines for which the stream function increases by 0.1. The superposition of both flows, the undisturbed c_0 (fig. 12, dotted) and the deflection flow (fig. 12, thin solid) can thus be composed by known methods to get the resultant flow (fig. 12, solid). Similarly the potential lines can be obtained. Since, in general, only a few points need be taken, it is simple to determine them by vector addition.

At the top of figure 13 the blade is shown for the undisturbed flow with its streamline and potential-line grid. If because of the adjacent blades the grid is distorted (fig. 13, bottom), the corresponding point of the blade, that is, leading or trailing edge lies at the point having the same potential and stream function (conformal transformation).

INFLUENCE OF FINITE ADJACENT BLADE DIMENSIONS

For these calculations we have calculated the influence of the deflection resulting from replacing the adjacent blades by a single vortex placed at the center of pressure of the blade. In most cases this approximation is completely satisfactory. Now if the blade chord is large compared to the spacing $a \cos \beta$, it should be noted that the space distribution of circulation in the adjacent blades must be considered. The vortex distribution along a blade chord is at least approximately known from lift and pressure distribution. Figure 14 shows the lift distribution (or the identical circulation distribution $\frac{\partial \Gamma}{\partial s}$) about our profile. It the profile is distorted by conformal transformation, each part is itself deformed. The circulation about an element of the profile remains unchanged. Therefore, the circulation for each unit length must diminish proportionally to the stretching and conversely, as the profile is deformed. Because of this displacement of the circulation, the center of pressure is itself somewhat shifted. The computations can then be carried out by shading the entire profile; the change, however, is relatively unimportant as shown by figure 17 (thin broken and solid lines). On the basis of the assumed circulation distribution given in figure 14, the new distribution and center of pressure given by the distortion is shown in figure 15. We proceed on the basis that the concentrated vortex at the center of pressure can be replaced by an appropriate vortex at each element on the blade. The influence for the vortex series at each element can then be separated and summed. Instead of working with finite elements, we can change to the differential ds, for which the circulation is $\frac{\partial \Gamma}{\partial s}\,ds$ and find the total deviation by integration along the chord. The latter method will be used in the example.

There is, in general, only a very small correction to the calculations carried out for the concentrated vortex. It is thus almost completely satisfactory to compute only the correction for the leading and trailing edges of the profile obtained from equations (9) and (13). To illustrate the method for obtaining the calculations at the trailing edge, we choose about 4 points for the blade (a, b, c, d, fig. 15) and determine from charts 6 and 7 the effect that a vortex series, of unit circulation at this point, has for the time being on the velocity at the trailing edge. If the point has coordinates ξ and η with respect to the zero point of the airfoil in question, and the trailing edge, the coordinates x, y, we thus find from our chart the disturbance due the vortex series at the point $\frac{x-\xi}{a}$ $\frac{y-\eta}{a}$ (fig. 16). We thus obtain the two components u and v of the disturbance velocity at the trailing edge of the blade, (fig. 15, curve E_u for the u component and E_v for the v component) and multiplying the value of the distribution curve by these influence lines, we thus get two curves which give the deviation due the distributed vortex at the blade point (fig. 15 $E_u \frac{\partial \Gamma}{\partial s} \frac{t}{\Gamma}$ and $E_v \frac{\partial \Gamma}{\partial s} \frac{t}{\Gamma}$). The total deflection at the trailing edge of the blade distributed vortex is obtained by planimetering the curves.

$$u = 0.34 \frac{\Gamma}{a}$$

$$v = 0.14 \frac{\Gamma}{a}$$

The concentrated vortex at the center of pressure gives from charts 6 and 7,

$$u' = 0.43 \frac{\Gamma}{a}$$

$$v' = 0.15 \frac{\Gamma}{a}$$

For the last values equation (9) gives for the angular deflection,

$$\Delta\beta' = \frac{v' \cos \beta - u' \sin \beta}{c_o + u' \cos \beta + v' \sin \beta} = - 6\tfrac{1}{4}^{\circ} \qquad (14)$$

as in figure 13. On the basis of the distributed vortex we get

$$\Delta\beta = \frac{u \sin \beta - v \cos \beta}{c_o + u \cos \beta + v \sin \beta} = - 4\tfrac{1}{2}^{\circ} \qquad (15)$$

We must therefore, for the vortex distributed along the blade chord, consider an increase in the exit angle of

$$\Delta\beta - \Delta\beta' = - 4\tfrac{1}{2}^{\circ} + 6\tfrac{1}{4}^{\circ} = 1\tfrac{3}{4}^{\circ}$$

In general this small angular difference will be given with sufficient accuracy by the following approximate equation

$$\Delta\beta - \Delta\beta' = \frac{(v - v') \cos \beta - (u - u') \sin \beta}{c_o + u' \cos \beta + v' \sin \beta} \qquad (16)$$

The disturbance component in the small direction c_o for the concentrated vortex is

$$\left.\begin{aligned} &u' \cos \beta + v' \sin \beta = 0.29\ c_o \\ &\text{and for the distributed vortex} \\ &u\ \cos \beta + v \sin \beta = 0.24\ c_o \end{aligned}\right\} \qquad (17)$$

We must therefore displace outward the trailing edge in the direction c_o from that given by equation (13) by a distance

$$\Delta s_A = 0.02a \qquad (18)$$

The corresponding calculation for the leading edge gives a change in angle of

$$\Delta\beta - \Delta\beta' = -\tfrac{3}{4}^{\circ} \qquad (19)$$

and a displacement from the zero point of a distance

$$\Delta s_E = 0.025a \qquad (20)$$

The profile with these changes is shown in figure 17. For comparison the profile from the assumed concentrated vortex is shown in thin lines and the profile for changed center of pressure due to distribution is shown in thin dotted lines. Thus the effect of distribution is seen to be fairly unimportant.

The example was intentionally chosen so the various influences had discernable effect for the various practicable applications illustrated by the diagrams. In general, it is soon obvious which method must be used to obtain the profile with sufficient accuracy. Excessive accuracy for many cases has little sense as the profile properties for strongly curved streams and especially for large pressure drop or increase through grid are themselves not accurate.

(See footnote, p. 2, K. Christiani.) In many cases a simple angular correction from equation (11) is sufficiently accurate.

The curved blade can naturally be obtained if the effect of a blade grid of given shape is estimated. The deviation velocity is obtained, from an estimated circulation about the blade, from the shape and angle of attack of the blades in an undisturbed flow. By repeating these calculations for the circulation, agreement with the desired value can be quickly obtained.

Translation by J. W. McBride,
Massachusetts Institute of
Technology.

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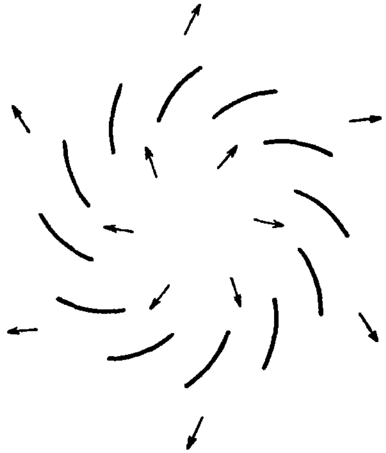

Figure 1.- Airfoil arranged in circle.

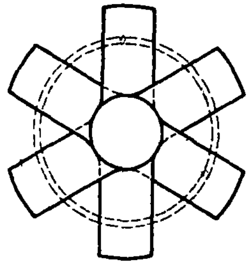

Figure 2.- Cylindrical intersection through propeller wheel.

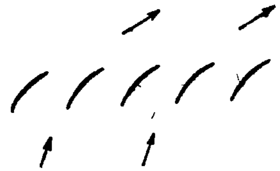

Figure 3.- Airfoils arranged in straight line.

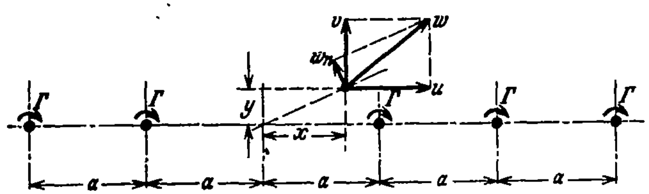

Figure 4.- Explanation of symbols. w- velocity at point xy; u,v,w_n components of same.

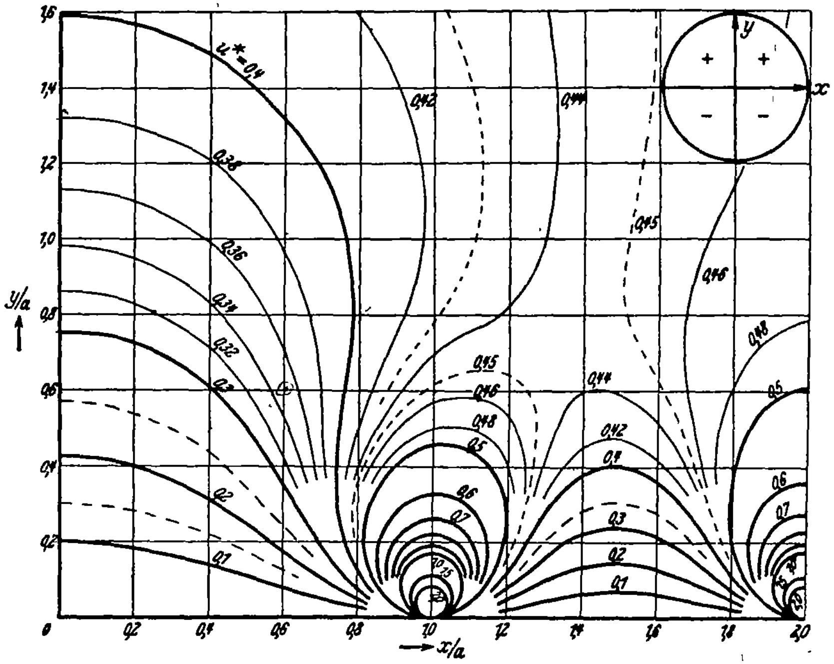

Figure 6.- Velocity component $u^*=u\frac{a}{\Gamma}$ parallel to lattice direction.

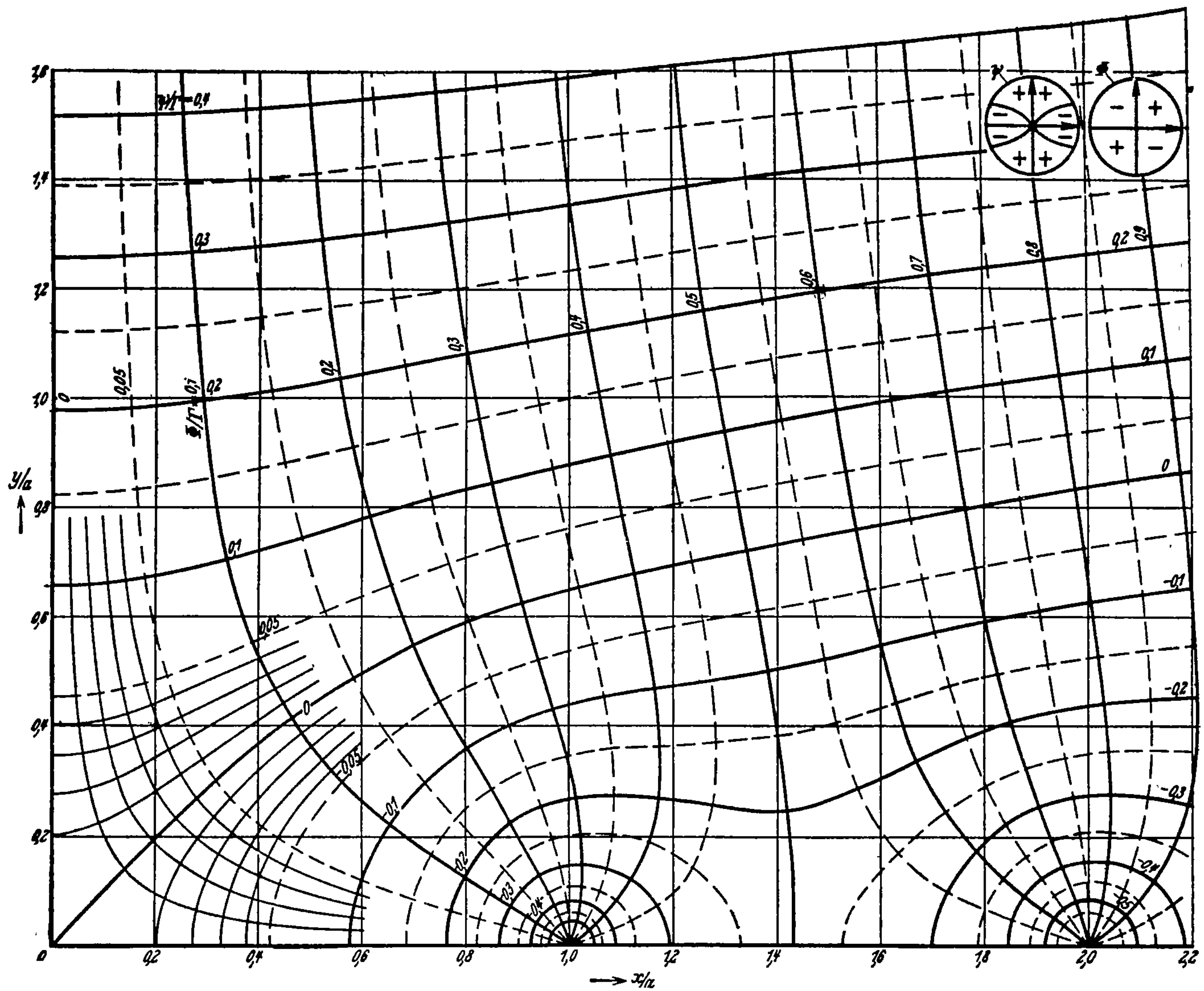

Figure 5. Potential Φ and stream function ψ.

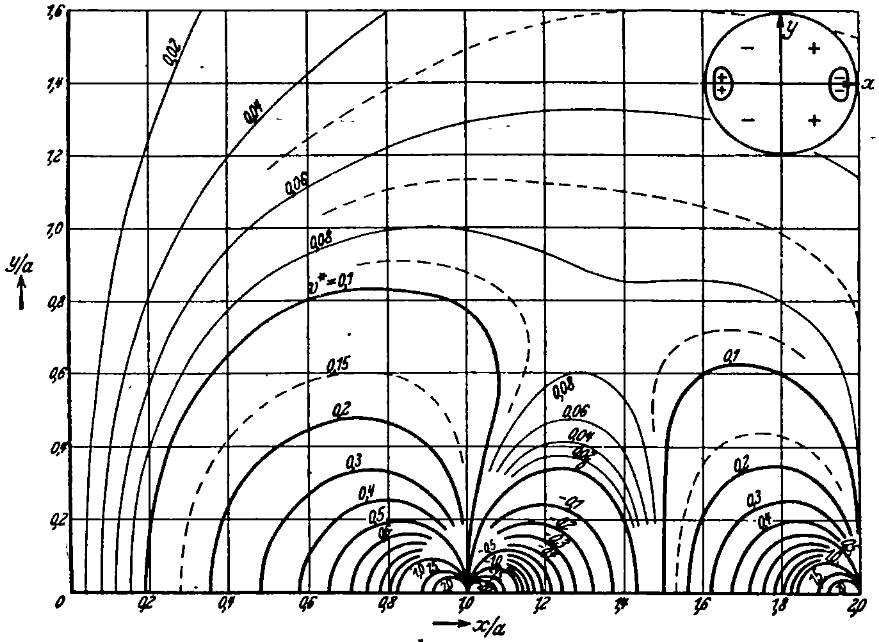

Figure 7.- Velocity component $v^*=v\frac{a}{\Gamma}$ normal to lattice direction.

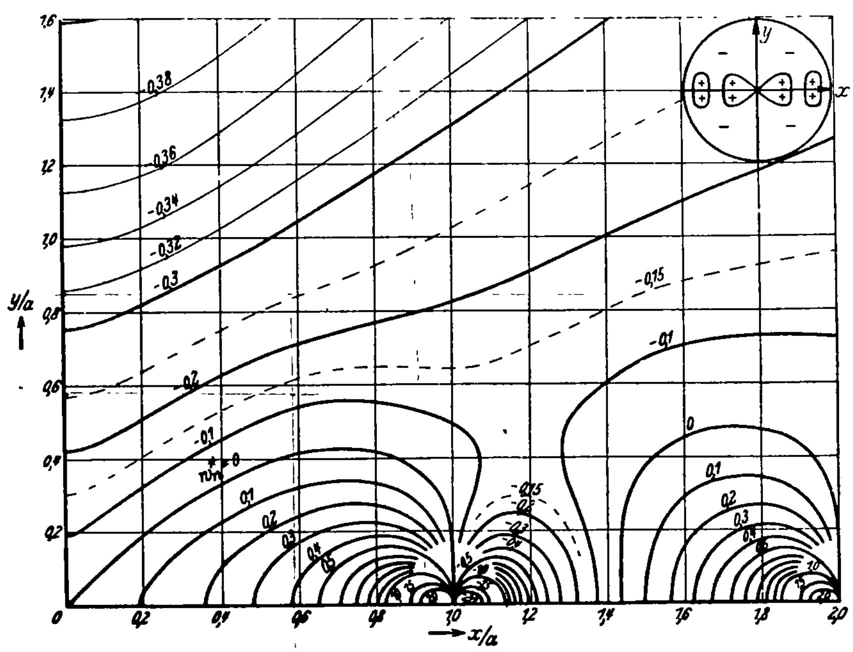

Figure 8.- Velocity component $w^*_n=w_n\frac{a}{\Gamma}$ normal to radius.

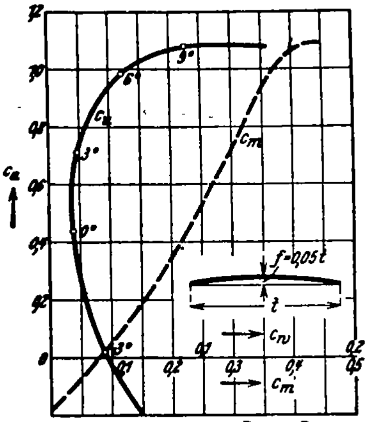

Figure 9.- Lift,drag, and moment of an arc shaped, cambered plate of infinite aspect ratio.

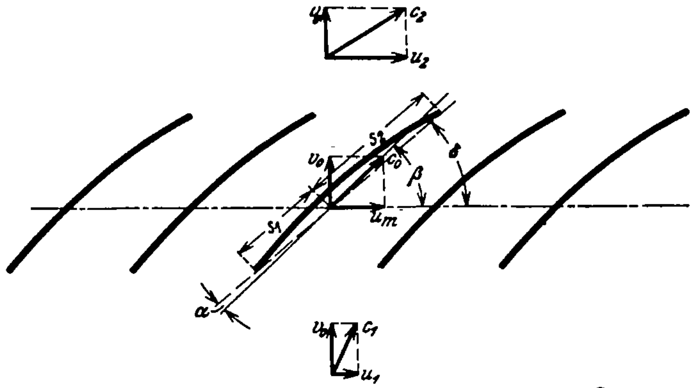

Figure 10.- Form and setting of airfoils in first approximation(taking no account of the effect of neighboring airfoils).

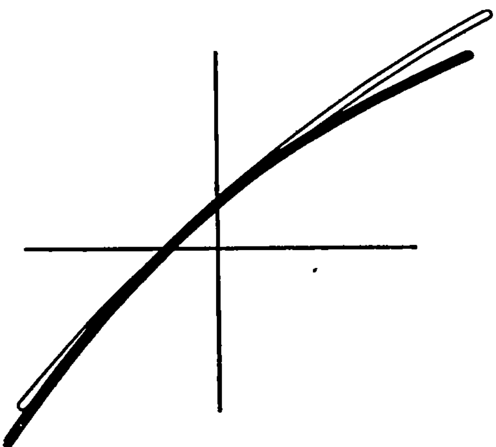

Figure 11.- Effect of disturbance velocities on the profile shape (second approximation). Light line shows profile of first approximation.

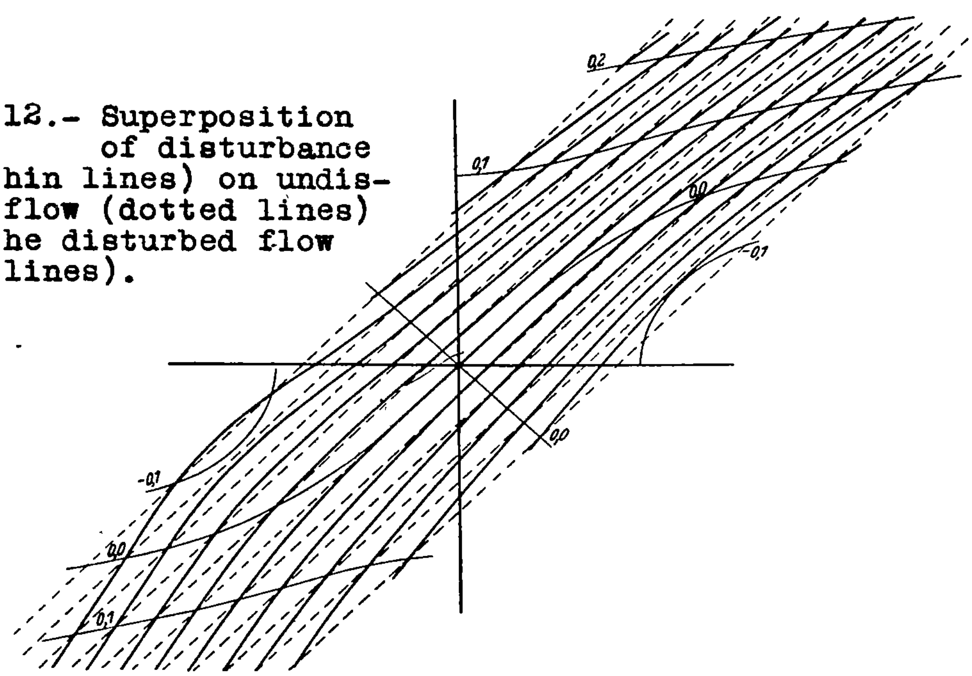

Figure 12.- Superposition of disturbance flow (thin lines) on undisturbed flow (dotted lines) gives the disturbed flow (heavy lines).

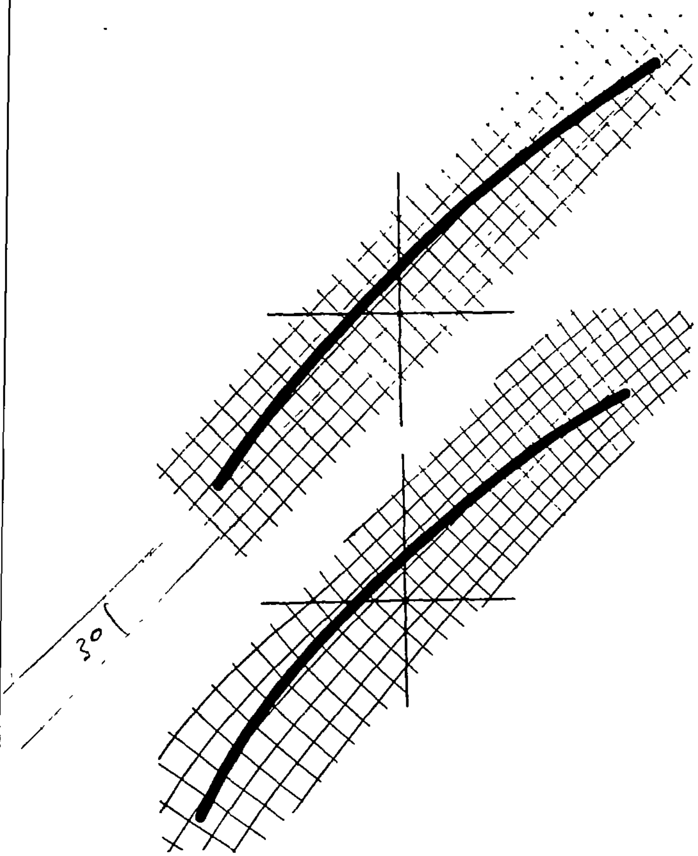

Figure 13.- Conformal transformation of profile in the undisturbed flow (top) on the profile in the disturbed flow (bottom) (second approximation).

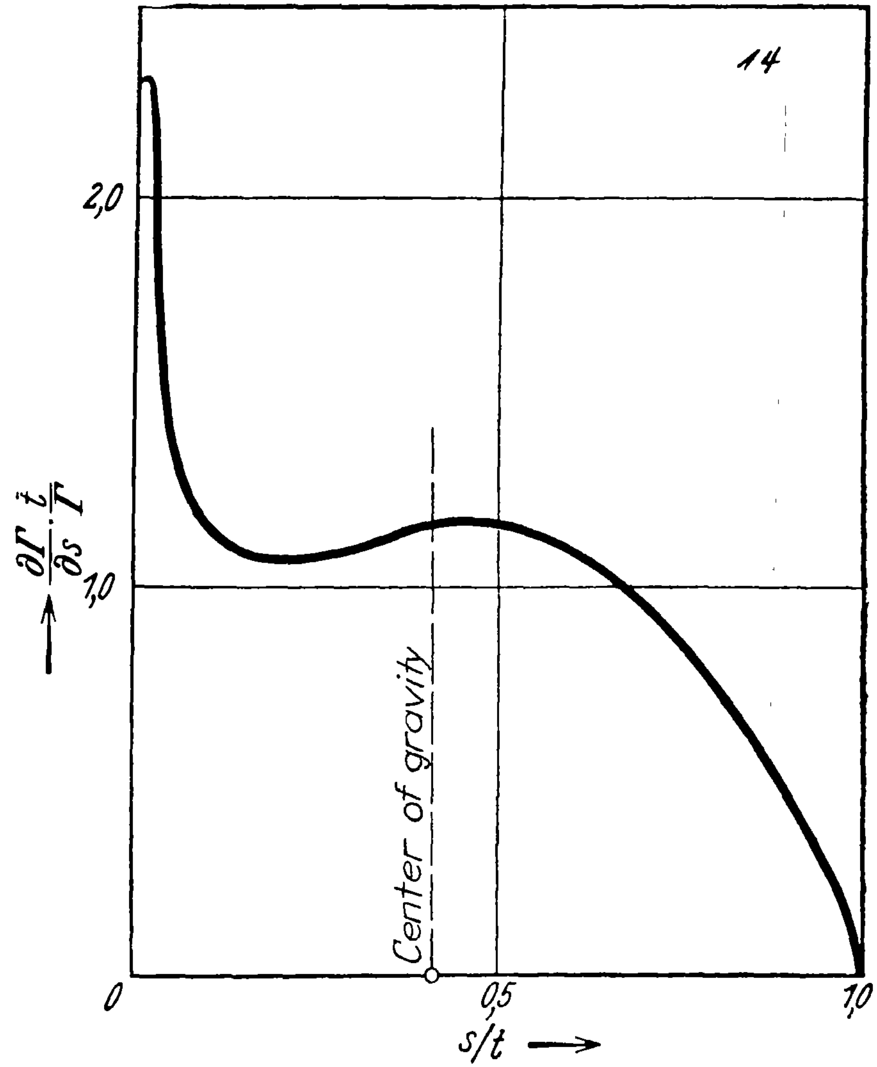

Figure 14.- Approximate distribution of the circulation over the airfoil in undisturbed flow.

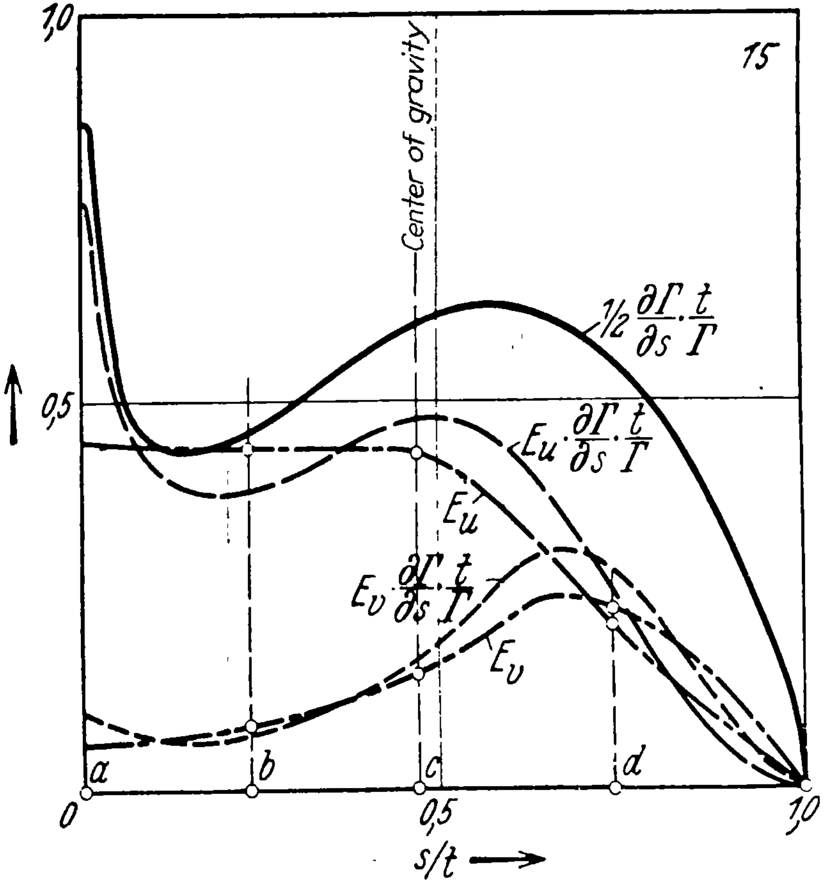

Figure 15.- Distribution of circulation over disturbed airfoil and its effect on the trailing edge.

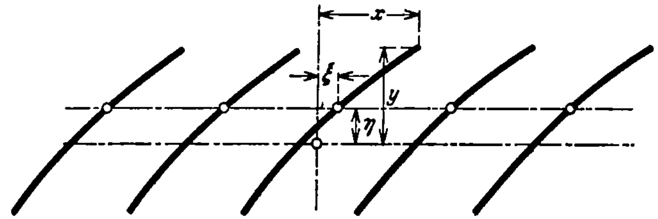

Figure 16.- Position of vortex at trailing edge of airfoil.

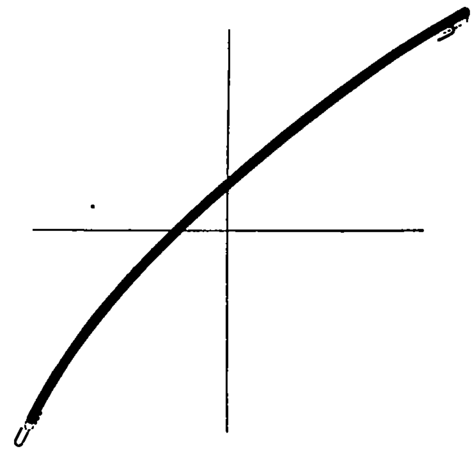

Figure 17.- Profile shape with vortex distribution over the airfoil taken into account (third approx.). Thin line shows profile of second approx. (according to Fig. 13) for comparison. Dotted line shows position of profile after displacement due to displacement of center of gravity.